for the equation to the tangent to the hyperbola,

$$xx' - p(x'y + y'x) - yy' = c(1 - m)$$

or

$$y = \frac{x' - py'}{y' + px'}\, x - \frac{c(1 - m)}{y' + px'}. \tag{6}$$

Let α equal the angle between the central radius and the tangent to the ellipse, and β equal the angle between the tangents to the ellipse and the hyperbola; then

$$\tan\beta = \frac{x'y' - py'^2 + mx'y' + mpx'^2}{y'^2 + px'y' - mx'^2 + mpx'y'},$$

$$\tan\alpha = \frac{mx'^2 + y'^2}{(m - 1)x'y'}.$$

That $\tan\alpha$ may equal $\tan\beta$, we must have, discarding the primes,

$$(mx^2 + y^2)(y^2 + pxy - mx^2 + mpxy) = (mxy - xy)(xy - py^2 + mxy + mpx^2),$$

which reduces to

$$y^4 + 2mpx^3y + 2mpxy^3 - m^2x^4 - m^2x^2y^2 + x^2y^2 = 0, \tag{7}$$

or, substituting for y its value from (1),

$$m^2(c - x^2)^2 + 2mpx^3\sqrt{m(c - x^2)} + 2m^2px(c - x^2)\sqrt{m(c - x^2)},$$
$$- m^2x^4 - m^3x^2(c - x^2) + mx^2(c - x^2) = 0. \tag{8}$$

From (3),

$$2px\sqrt{m(c - x^2)} = (1 + m)x^2 - c.$$

Substituting in (8), we get

$$m^2(c - x^2)^2 + mx^2(x^2 + mx^2 - c) + m^2(c - x^2)(x^2 + mx^2 - c) - m^2x^4 - m^3cx^2$$
$$+ m^3x^4 + mcx^2 - mx^4 = 0,$$

an identity. Therefore $\tan\alpha = \tan\beta$, and $\alpha = \beta$. Q. E. D.

[*S. M. Barton.*]

EXERCISES.

13

A THIN inextensible cord in which the density of the material increases in geometric progression, as the distance from one end increases in arithmetic pro-

gression is laid directly across a rough horizontal cylinder the circumference of a normal section of which is equal to twice the length of the cord; to determine the co-efficient of friction, supposing the cord to be only just supported when its extremities are in the horizontal plane containing the axis of the cylinder. [*William Hoover.*]

14

A FAISCEAU of parabolas can be drawn having the pole of a cardioid as the common focus, all passing through one point and all cutting the cardioid at right angles. [*H. A. Newton.*]

15

IT IS required to inscribe a rectangle of breadth nD within another rectangle whose diagonal is B. Show that the excess E of the declivity M of the diagonal of the greater rectangle over that of the length of the less may be calculated when n is small, from the formula

$$\sin E = n \cos 2M [1 + 2n \sin 2M + n^2 (3 - \cos 2M) + \ldots].$$

[*W. M. Thornton.*]

16

ELIMINATE x from the equations

$$x^6 + ax^2 + bx + c = 0,$$
$$x^3 + dx^2 + ex + f = y,$$

and arrange the result according to powers of y. [*Mansfield Merriman.*]

17

IN HIS work, *Die lineale Ausdehnungslehre, ein neuer Zweig der Mathematik*, p. 65, Grassmann says: "Lagrange führt in seiner *Méc. Anal.*, p. 14 der neuen Ausgabe, einen Satz von Varignon an, dessen er sich zur Verknüpfung der verschiedenen Principien der Statik bedient. * * * * Dieser Satz ist, wie sich sogleich zeigen wird, unrichtig."

In what way is this theorem incorrect as used by Todhunter and others?

[*Asaph Hall.*]

18

A FIXED point of light shines upon a plane which rotates about a fixed axis with a constant angular velocity. A normal to this plane moves at a constant rate (measured in the plane) in the direction of the shadow which the normal casts upon the plane. Given the position of the normal at a time when the light is in the plane; when the light is again in the plane, how far will the normal have moved during the intervening time? [*F. P. Leavenworth.*]